

A grayscale photograph of the main building of Leibniz University Hannover, featuring a prominent central tower and a large equestrian statue in the foreground.

# Estimators for Meta-Analysis

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# Introduction

## How to conduct a meta-analysis

### Helpful literature

Stanley, 2001, Wheat from Chaff: Meta-Analysis as a Quantitative Literature Review, JEP, 131-150.

Stanley et al., 2013, Meta-analysis of economics reporting guidelines, JES, 390-394.

Stanley/Doucouliagos, 2012, Meta-Regression Analysis in Economics and Business, Oxford: Routledge.

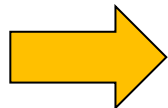
Nelson/Kennedy, 2009, The use (and abuse) of meta-analysis in environmental and natural resource economics: an assessment, Environ. Resource Econ., 345-377.

Feld/Heckemeyer, 2011, FDI and taxation: a meta study, JES, 233-273.

Step #1: Include all relevant studies from a standard database

Step #2: Choose a summary statistic and reduce the evidence to a common metric

Step #3: Choose moderator variables

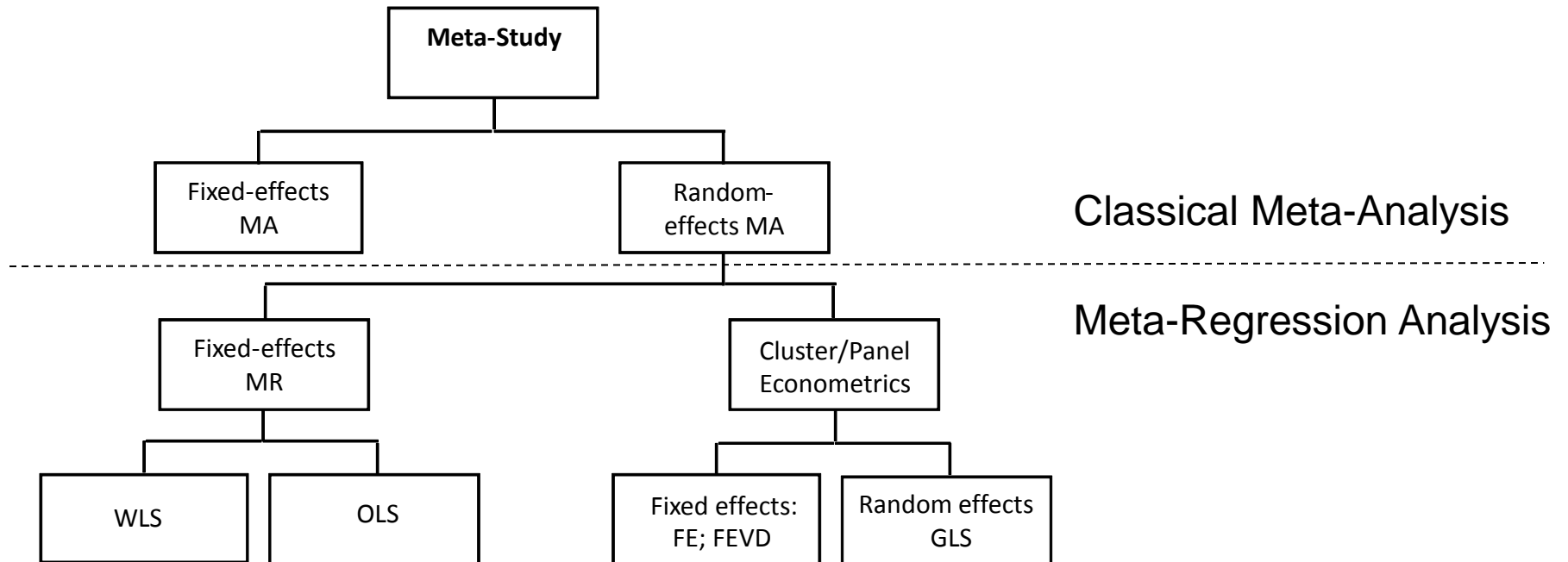


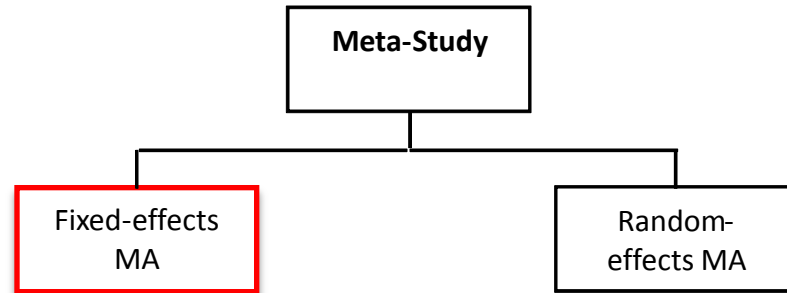
Step #4: Conduct a meta-regression analysis

# Step #4: Conduct a meta-regression analysis

# Meta-Analysis

## A Schema of Meta-Analytical Estimators





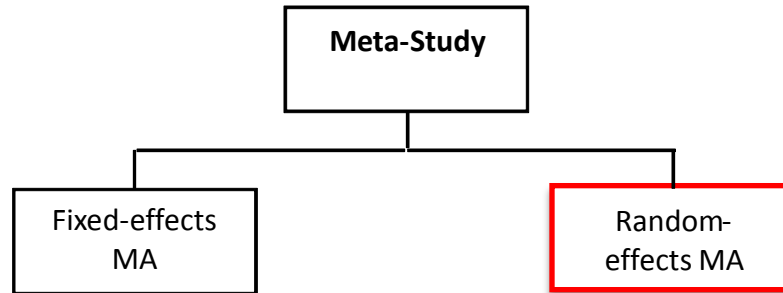
## Classical Meta-Analysis

### Fixed-effects meta-analysis

$$\hat{\gamma}_i = \gamma_0 + \varepsilon_i \quad (1)$$

Main assumption: no heterogeneity

- Every primary study produces a single unbiased estimate of the unknown elasticity value
- Each study has been conducted in a similar fashion (no impact of design features)
- Estimates are stochastically independent of each other



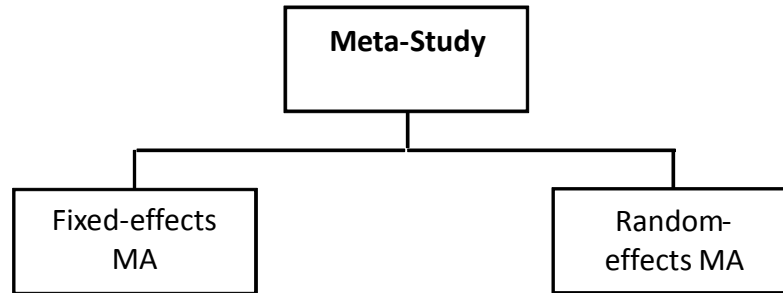
## Classical Meta-Analysis

### Random-effects meta-analysis

$$\hat{\gamma}_i = \gamma_0 + \mu_i + \varepsilon_i \quad (2)$$

Main assumption: unexplainable heterogeneity

- True elasticity is not assumed fixed but with random component
- Primary estimates differ beyond pure sampling error
- Rarely encountered in economics because sources of heterogeneity are usually apparent and testable



## Classical Meta-Analysis

### Estimation

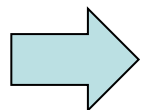
$$\bar{\gamma}_i = \frac{\sum_{i=1}^L w_i \hat{\gamma}_i}{\sum_{i=1}^L w_i}$$

$$w_i = 1/V(\varepsilon_i)$$

in fixed-effects MA

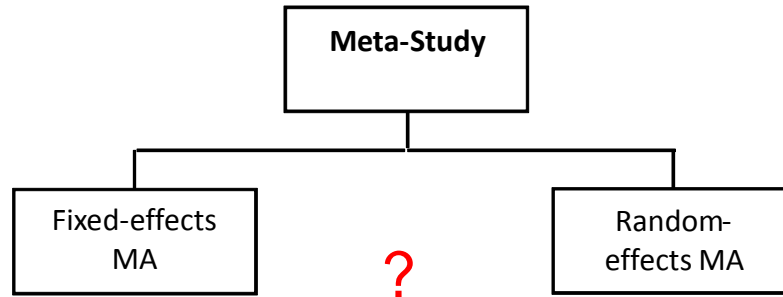
$$w_i = 1/(V(\varepsilon_i) + \sigma_\mu^2)$$

in random-effects MA



it's Weighted-Least-Squares (WLS) of equ. (1) or (2)  
with analytical weights  $w_i$





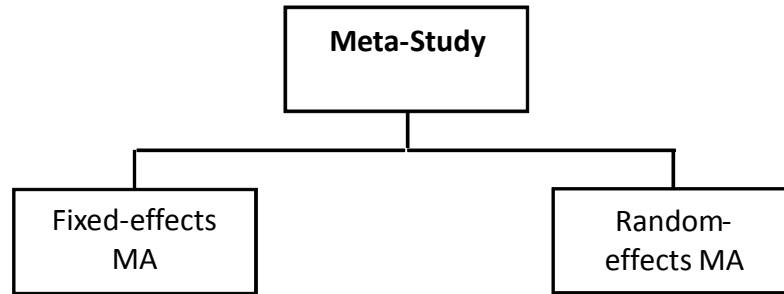
## Classical Meta-Analysis

### The Q-test

$$Q = \sum_{i=1}^L w_i \hat{\gamma}_i^2 - \frac{\left( \sum_{i=1}^L w_i \hat{\gamma}_i \right)^2}{\sum_{i=1}^L w_i} \sim \chi^2(L - 1)$$

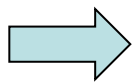
# Meta-Analysis

## Classical Meta-Analysis



Classical Meta-Analysis

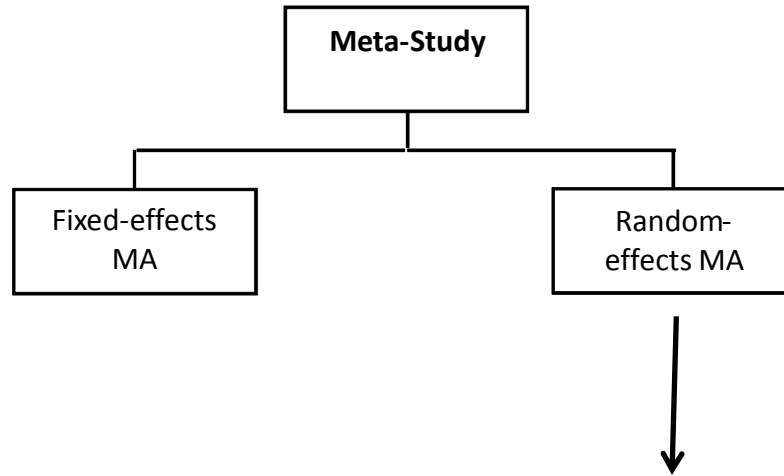
Stata



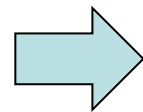
metan

# Meta-Analysis

## Meta-Regression Analysis



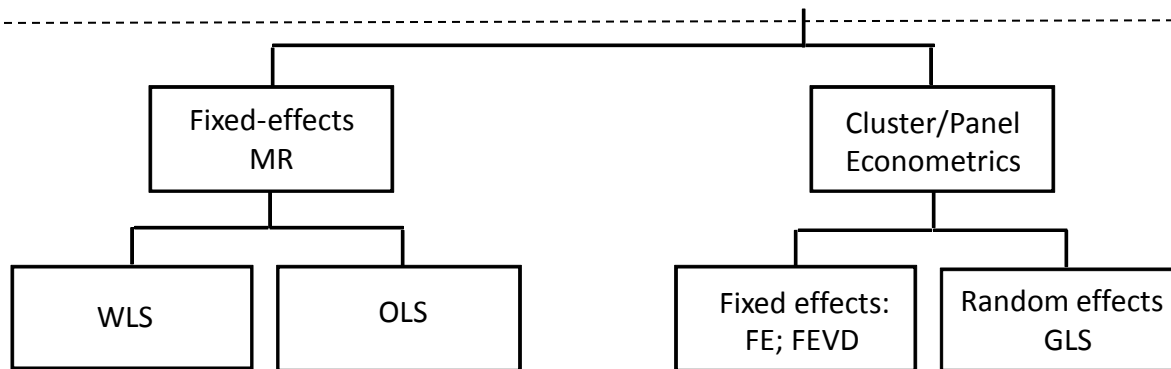
If there is heterogeneity, explain it!



Meta-regression

# Meta-Analysis

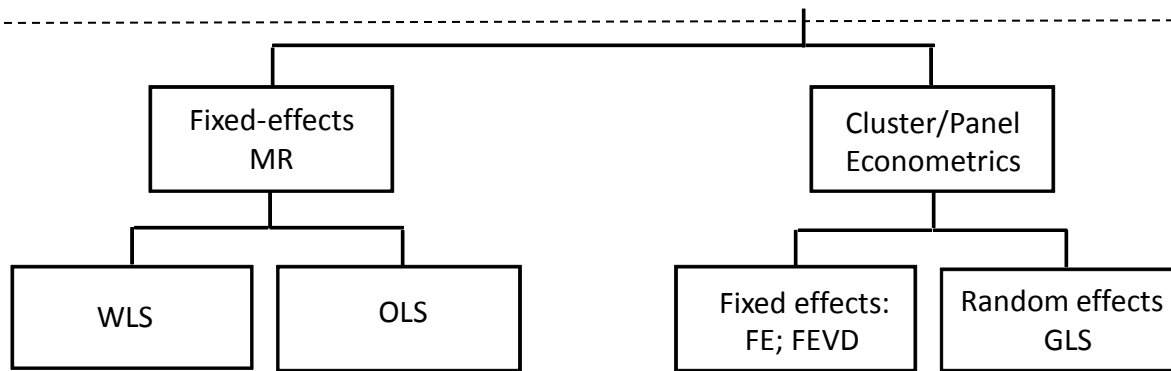
## Meta-Regression Analysis



## Meta-Regression Analysis

Regression models estimated	No. of studies
OLS with or without SE corrections	98
Weighted least squares	36
Panel/Multilevel models	29
Other	38

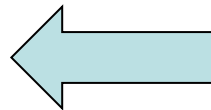
Nelson/Kennedy (2009)



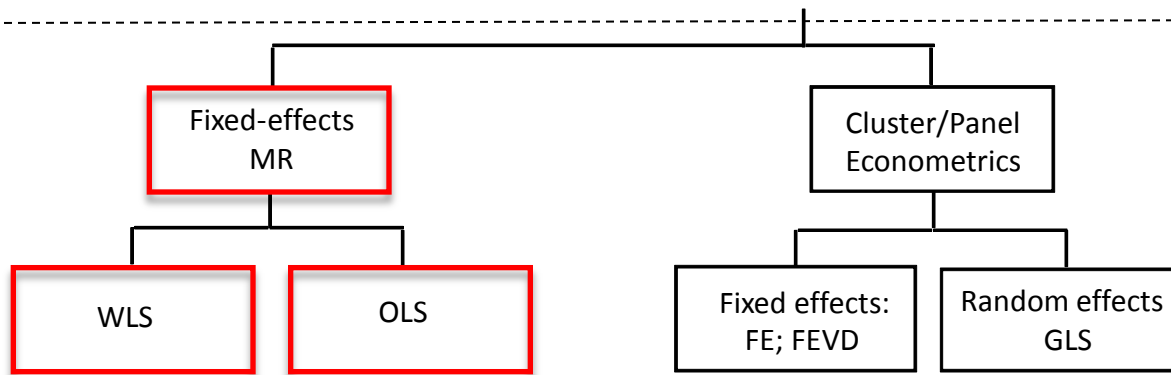
## Meta-Regression Analysis

### 2 issues

1) Heteroscedasticity



2) Lack of independence of observations and unobserved study effects



## Meta-Regression Analysis

### Fixed-effects meta-regression

$$\hat{\gamma}_i = \gamma_0 + \mathbf{x}_i \boldsymbol{\beta} + \varepsilon_i \quad (3)$$

$$\sigma_{\varepsilon_i}^2 = V(\hat{\gamma}_i | \mathbf{x}_i) \quad \longrightarrow \quad \text{varies across studies } i$$

$\longrightarrow$  but we have (good?) information about  $V(\hat{\gamma}_i | \mathbf{x}_i)$

# Meta-Analysis

## Meta-Regression Analysis

Table 8

The impact of taxes on the financial leverage of subsidiaries of multinational firms

The dependent variable in columns (1) to (3) is the ratio of subsidiary total liabilities to subsidiary total assets (financial leverage). The dependent variable in column (4) is the ratio of total liabilities minus accounts payable minus cash to total assets minus accounts payable minus cash (adjusted financial leverage). Effective marginal tax rate,  $\tau$ , is the statutory tax rate on dividend income generated in the subsidiary country, taking withholding taxes and the tax system for foreign source income into account. Tax incentive to shift debt is the sum of international tax differences between subsidiary countries weighted by subsidiary asset shares, taking withholding taxes and the international tax system into account. Tangibility is the ratio of subsidiary fixed assets to subsidiary total assets. Log of sales is the logarithm of subsidiary sales. Profitability is the ratio of subsidiary earnings before interest, taxes, depreciation, and amortization to subsidiary total assets. Creditor rights is the index of country creditor rights from Djankov, McLiesh, and Shleifer (2007). Political risk is the index of political risk from *International Country Risk Guide*. We inverted the scale with higher scores indicating greater risk. Inflation is the annual percentage change in CPI of the subsidiary's host country. Growth opportunities is the median of the annual growth rate of subsidiary sales in a subsidiary's country and industry. Sample consists of subsidiaries of European companies in Amadeus for the period 1994–2003. All regressions are estimated using OLS and include parent, industry, and year fixed effects. We report White (1980)'s heteroskedasticity-consistent standard errors between brackets. \* denotes significance at 10%; \*\* significance at 5%; and \*\*\* significance at 1%.

Variables (predicted sign of coefficient)	(1) Financial leverage	(2) Financial leverage	(3) Financial leverage	(4) Adjusted financial leverage
Effective marginal tax rate (+)	0.259*** (0.017)	0.162*** (0.031)	0.184*** (0.033)	0.195*** (0.044)
Tax incentive to shift debt (+)		0.132*** (0.031)	0.120*** (0.033)	0.178*** (0.045)
Tangibility (+/-)	-0.130*** (0.005)	-0.123*** (0.006)	-0.120*** (0.006)	0.105*** (0.008)
Log of sales (+)	0.022*** (0.001)	0.023*** (0.001)	0.023*** (0.001)	0.022*** (0.001)
Profitability (+/-)	-0.062** (0.025)	-0.055** (0.027)	-0.060* (0.032)	-0.081* (0.042)
Creditor rights (+)			0.006*** (0.001)	0.019*** (0.002)
Political risk (+/-)			0.001*** (0.000)	-0.000 (0.000)
Inflation (-)			-0.001*** (0.000)	-0.002*** (0.000)
Growth opportunities (+)			0.021*** (0.008)	0.010 (0.009)

$\hat{y}_i$

$\sqrt{\hat{V}(\hat{y}_i | x_i)}$

Source: Huizinga/Laeven/Nicodeme (2008)

If information on  $V(\hat{\gamma}_i | \mathbf{x}_i)$  is **good/adequate**, **WLS** with precision weights is much **more efficient** than OLS.

If information on  $V(\hat{\gamma}_i | \mathbf{x}_i)$  is **not** good/adequate, **WLS** with precision weights is **inefficient**.

⇒ see, e.g., Greene, Econometric Analysis, 2012: p. 318f.



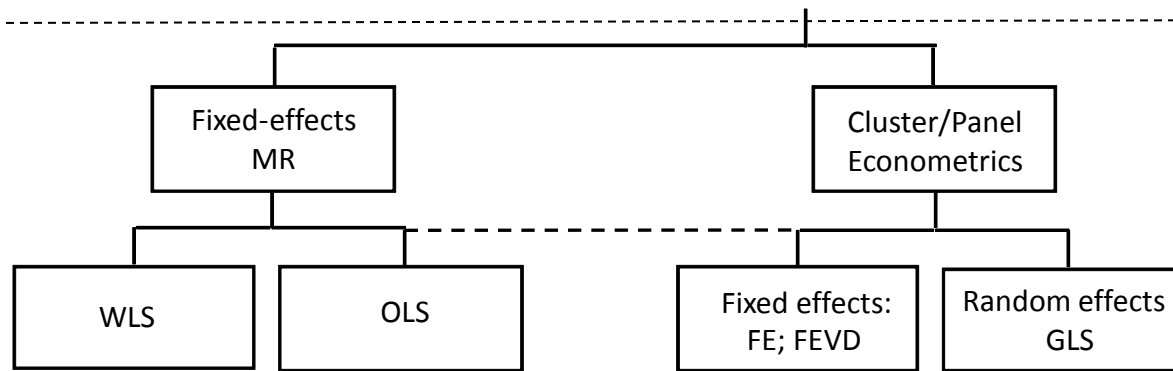
Threats to the quality of info on  $V(\hat{\gamma}_i | \mathbf{x}_i)$  (examples):

- 1) Reported standard errors are not robust to heteroscedasticity or clustering of observations although this would be required

➡ anti-conservative inference, i.e. too small standard errors

- 2) Interaction terms in primary literature

➡ standard errors of joint effects must be combined using the delta method, but generally no information on covariance between base coefficient and coefficient of the interaction term

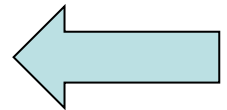


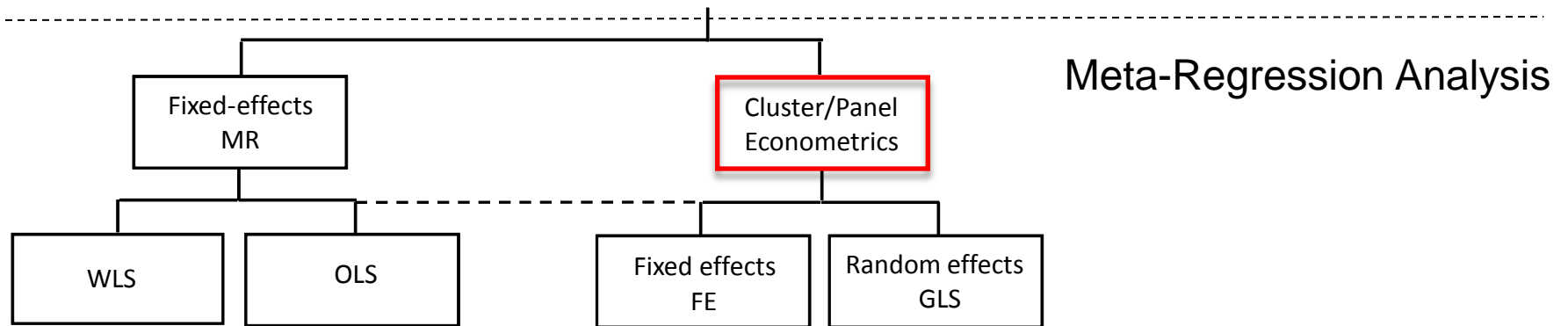
## Meta-Regression Analysis

### 2 issues

1) Heteroscedasticity

2) Lack of independence of observations and unobserved study effects



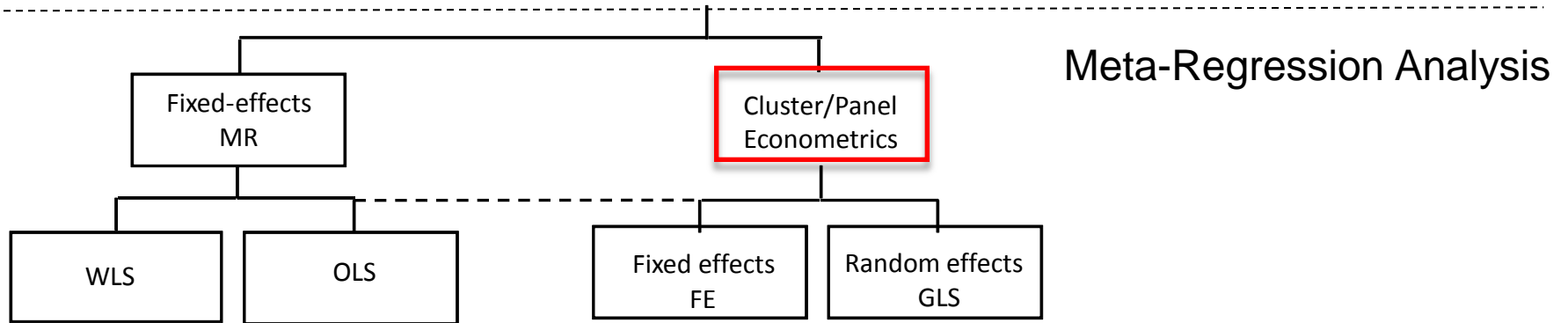


### 2 issues

- 1) Heteroscedasticity
- 2) Lack of independence of observations and unobserved study effects

➡ Multiple estimates are sampled from individual studies

➡ Multiple estimates are sampled from the same author(s)



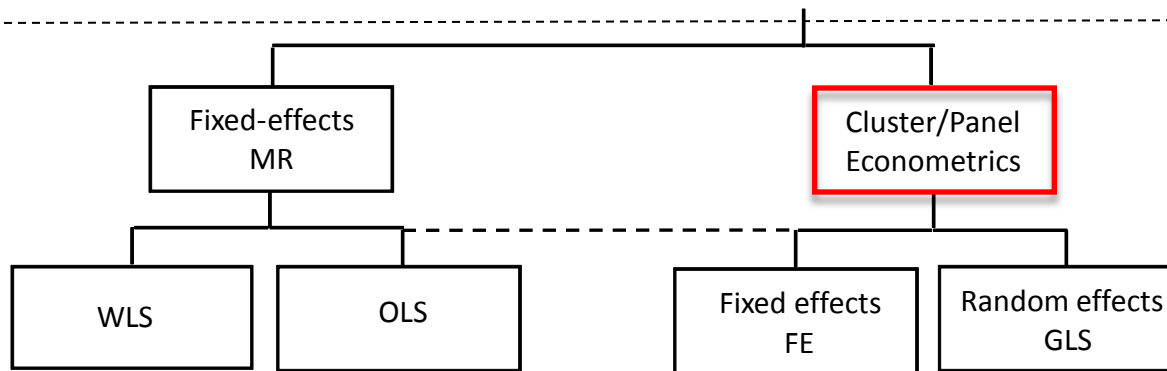
Study	Estimate
1	0.54
1	0.55
1	0.59
1	0.53
2	0.25
2	0.26
2	0.23
3	0.34
3	0.35

### Clustered/Panel meta-regression

$$\hat{\gamma}_i = \gamma_0 + \mathbf{x}_i \boldsymbol{\beta} + \mu_i + \varepsilon_i$$



Unobserved study-specific effects



## Meta-Regression Analysis

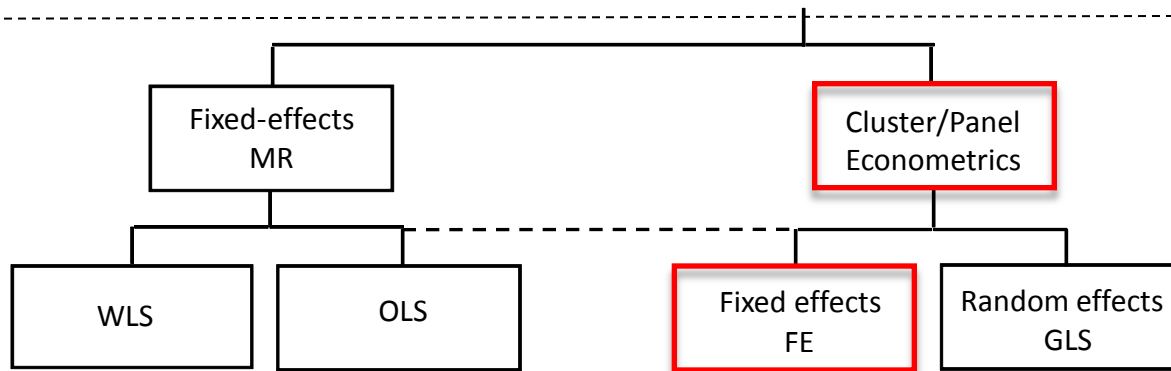
### Clustered/Panel meta-regression

$$\hat{\gamma}_i = \gamma_0 + \mathbf{x}_i \boldsymbol{\beta} + \mu_i + \varepsilon_i$$

Unobserved study-specific effects

→ Creating within study-dependence

→ Potentially correlate with observable study characteristics!

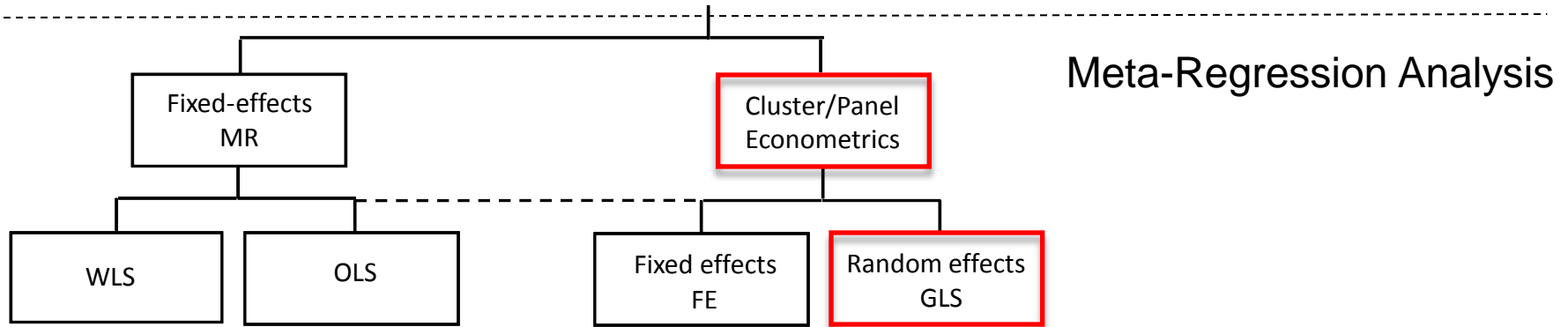


## Meta-Regression Analysis

### Clustered/Panel meta-regression

Alternative solution: Explicitly control for study-fixed effects by including study dummies/use a within estimator

- ➡ Advantages:
- consistent estimation
  - Lack of independence is taken into account
- ➡ Disadvantage: Study dummies absorb all between-study variation. Impact of study dimensions that vary between and never within studies (e.g. geographical scope) is unidentifiable.



### Clustered/Panel meta-regression

Alternative solution: Random effects estimation

➡ Advantages: dependence is taken into account

➡ Disadvantage: GLS weighting matrix ignores information about  $V(\hat{y}_i | \mathbf{x}_i)$

$$\begin{pmatrix} \sigma_c^2 + \sigma_u^2 & \sigma_c^2 & \dots & \sigma_c^2 \\ \sigma_c^2 & \sigma_c^2 + \sigma_u^2 & \dots & \vdots \\ \vdots & \dots & \ddots & \sigma_c^2 \\ \sigma_c^2 & \dots & \dots & \sigma_c^2 + \sigma_u^2 \end{pmatrix}$$

# Meta-Analysis

## Meta-Regression Analysis

	Good information about $V(\hat{\gamma}_i   \mathbf{x}_i)$	No or bad information about $V(\hat{\gamma}_i   \mathbf{x}_i)$
No cluster sample	WLS	OLS with heteroscedasticity- robust SE
<i>Stata</i>	<i>reg with option aweight [1/V(<math>\hat{\gamma}_i   \mathbf{x}_i</math>)]</i>	<i>Reg with option robust</i>
Cluster sample	WLS with cluster-robust SE (and check robustness with random effects GLS)	Random effects GLS with heteroscedasticity-robust (or cluster-robust) SE
<i>Stata</i>	<i>reg with option aweight [1/V(<math>\hat{\gamma}_i   \mathbf{x}_i</math>)] and cluster</i>	<i>xtreg with option robust or cluster</i>

Note: As problems of heteroscedasticity and lack of independence affect the efficiency of meta-estimates, but not their consistency, the choice of estimator should matter primarily for inference, less so for coefficient sign and magnitude.

The problem of study-specific unobservables remains, as long as no panel FE estimator is employed!!